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(b) State - Space Analysis:

(1) It is a time domain analysis, more accurate than transfer func.

(2) It provides an insight into the behaviour of the system.

(3) It can be applied to multiple i/p and o/p.

(4) It can be used in digital comp. sy.

(5) Application to all dynamic systems

State - The state of dynamic system at a time t_0 (or n_0) is defined as the minimal information that is sufficient to determine the state and the o/p of the system for all time $t \geq t_0$ (or $n \geq n_0$) when the i/p to the system is also known for all time $t \geq t_0$ (or $n \geq n_0$)

It is the group of variables which summarize the history of the system in order to predict future value.

State Vector: The N state variables may be considered as N component of a vector q. → state vector

$$q(t) = \begin{bmatrix} q_1(t) \\ \vdots \\ q_N(t) \end{bmatrix} \qquad q_n = \begin{bmatrix} q_1(n) \\ q_2(n) \\ \vdots \\ q_N(n) \end{bmatrix}$$

The N dimensional space whose co-ordinate axes consists of q_1 axis, q_2 axis ... q_N - axis is called state space.

State Space Representation of Continuous LTI system

Suppose a single-i/p continuous-time LTI system is →

$$\frac{d^N y(t)}{dt^N} + a_1 \frac{d^{N-1} y(t)}{dt^{N-1}} + \cdots a_N y(t) = x(t)$$

One set of initial condition is $y(0), y'(0), \ldots, y^{(N-1)}(0)$, $\quad y^{k}(t) = \frac{d^k y(t)}{dt^k}$

∴ N state variables $q_1(t), q_2(t) \ldots q_N(t)$

$$q_1(t) = y(t)$$
$$q_2(t) = y'(t)$$
$$\vdots$$
$$q_N(t) = y^{N-1}(t)$$

$$\therefore \dot{q}_1(t) = q_2(t)$$
$$\dot{q}_2(t) = q_3(t)$$

$$\therefore \dot{q}_N(t) = -a_N q_1(t) - a_{N-1} q_2(t) - \cdots - a_1 q_N(t) + x(t)$$

$$y(t) = q_1(t)$$

where $\dot{q}_k(t) = \frac{d q_k(t)}{dt}$

state variable N

$$\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \vdots \\ \dot{q}_N(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & & & \vdots \\ -a_N & -a_{N-1} & -a_{N-2} & -a_1 \end{bmatrix}}_{N \times N} \begin{bmatrix} q_1(t) \\ \vdots \\ q_N(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} x(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \cdots 0 \end{bmatrix} \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_N(t) \end{bmatrix}$$

$(N \times 1)$ matrix $q(t) = \begin{bmatrix} q_1(t) \\ q_2(t) \\ \vdots \\ q_N(t) \end{bmatrix}$

$$\frac{dq}{dt} = \dot{q}(t) = \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \vdots \\ \dot{q}_N(t) \end{bmatrix}$$

$$\therefore \dot{q}(t) = Aq(t) + bx(t)$$

$$y(t) = cq(t)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ -a_N & -a_{N-1} & -a_{N-2} & -a_2 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad c = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} \dot{q}(t) = Aq(t) + bx(t) \\ y(t) = cq(t) + dx(t) \end{cases}$$

~~A system is~~ described by differen eq.

Q. $\frac{d^3y(t)}{dt^3} + 6\frac{d^2y(t)}{dt^2} + 11\frac{dy}{dt} + 10y(t) = 8x(t)$ $\quad \dot{y}(t)$

where $y(t)$ is o/p

$x(t) \rightarrow$ i/p $\quad N=3$

$q_1 \; q_2 \; q_3$

find state space re...